Quintessence Models and the Cosmological Evolution of α.

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Abstract

The cosmological evolution of a quintessence-like scalar field ϕ coupled to matter and gauge fields leads to effective modifications of the coupling constants and particle masses over time. We analyze a class of models where the scalar field potential $V(\phi)$ and the couplings to matter $B(\phi)$ admit common extremum in ϕ, as in the Damour-Polyakov ansatz. We find that even for the simplest choices of potentials and $B(\phi)$, the observational constraints on $\Delta\alpha/\alpha$ coming from quasar absorption spectra, the Oklo phenomenon and Big Bang nucleosynthesis provide complementary constraints on the parameters of the model. We show the evolutionary history of these models in some detail and describe the effects of a varying mass for dark matter.

1 Introduction

The existence of dark energy supported by a number of cosmological observations remains one of the most outstanding puzzles in modern cosmology [1]. The cosmological constant and/or a quintessence field are the most commonly proposed candidates for dark energy. The possibility that a scalar field at early cosmological times follows an attractor-type solution [2, 3] and tracks the evolution of the visible matter-energy density while dominating the energy density in the Universe at late cosmological stages has been a subject of lively debates in the cosmology literature [4] - [20]. There are certain hopes that quintessence-like models may help alleviate the severe fine-tuning associated with the cosmological constant problem. This by itself represents a legitimate field of research and has triggered various model building efforts.

The most fundamental difference between quintessence and the cosmological constant is that the former represents a new very light degree of freedom, with a "wavelength" comparable to the size of the Hubble horizon. As a consequence, the dynamical scalar field gives rise to a time-dependent equation of state parameter, $\omega_\phi \equiv p_\phi/\rho_\phi$. Therefore, the variation of ω for dark energy with redshift is among the most important cosmological parameters to measure, and will be a large component of future efforts in cosmology.

Are there additional experimental ways of checking for the existence or absence of dark energy in the form of quintessence? Apart from "geometric" tests using standard cosmological methods (CMB, supernovae, lensing, etc.), one could also hope to detect the possible interaction of the quintessence field with matter. This can occur either through the observation of the effective change of the coupling constants and masses of particles over cosmological times, or via detecting additional components to the gravitational interaction due to exchange by the scalar. A positive detection of the cosmological change of coupling constants would be firm proof of the existence of new degree(s) of freedom with extremely long wavelength, thus providing a perfect candidate for quintessence.

For this reason, the recently reported indications of a change in the effective electromagnetic coupling constant [21], $\Delta\alpha/\alpha \sim -0.6 \times 10^{-5}$ at redshifts $z \sim 1$, if indeed true, can be considered as an independent detection of an ultra-light degree of freedom. These results [21] are based on the comparison of the relativistic shifts of atomic energy levels in quasar absorption spectra. For some time this result remained unchallenged, yet recently there have been attempts to detect a variation in α using similar methods [22] that has so far led to null results. Also, alternative (α-unrelated) explanations of the results of Ref. [21] based on a z-dependent Mg isotope abundance were shown to be astrophysically viable [23]. This rather controversial situation becomes even more complicated if other constraints on $\Delta\alpha/\alpha$ are taken into account [24]. The Oklo natural reactor and meteoritic abundances of rhenium provide stringent constraints on the change of the coupling constants that goes back

to $z \sim 0.1 - 0.4$ [25, 26, 27]. However, a recent re-analysis of Oklo phenomenon suggested that the present data are consistent with the non-zero change of $\Delta\alpha/\alpha = 4.5 \times 10^{-8}$ [28].

Despite the questionable status of the non-zero claim for $\Delta\alpha/\alpha$ there have been a number of attempts [14] - [17], [29] - [35] to build simple models that could account for a possible $O(10^{-5})$ relative shift in the fine structure constant at redshifts $z \sim 1$. It is widely accepted now that the minimal Bekenstein model [29] with a scalar field coupled *only* to the electromagnetic field in the initial Lagrangian,

$$\mathcal{L}_{\varphi F} = -\frac{1}{4} B_F \left(\frac{\varphi}{M_{\text{Pl}}} \right) F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} \left(1 + \zeta_F \frac{\varphi}{M_{\text{Pl}}} + \frac{\xi_F}{2} \left(\frac{\varphi}{M_{\text{Pl}}} \right)^2 + ... \right) F_{\mu\nu} F^{\mu\nu}, \qquad (1.1)$$

cannot provide $\Delta\alpha/\alpha$ larger than $10^{-9} - 10^{-10}$ level when the constraints on the nonuniversal gravitational interaction mediated by this field are imposed. This is because the electromagnetic portion of the baryon energy density that drives the cosmological evolution of φ is a very weak source. It has been suggested that the coupling to dark matter [30, 35, 33] and/or the self-interaction potential [34] of the scalar field drives its evolution. A number of tracker field and quintessence field models coupled to $F_{\mu\nu}^2$ have been analyzed with the universal conclusion that a linear coupling to the electromagnetic energy density on the order of $10^{-3} \lesssim \zeta_F \lesssim 10^{-5}$ can easily explain the QSO-suggested change of the fine structure constant and satisfy the experimental limits on the universality of the gravitational interaction. It has proven to be a harder task to have a substantial change in the fine structure constant at $z \sim 1$ and remain consistent with the Oklo and meteoritic constraints. Only a few models (see *e.g.* Refs. [15], [17]) are known to pass these requirements.

To this end, it will be useful to investigate a class of quintessence models where the potential $V(\varphi)$ admits a minimum at φ_0, and $\varphi(z)$ approaches φ_0 when $z \to 0$. Without loss of generality, we can put the extremum value to zero, and then $V(0) \simeq \Lambda = \Omega_\Lambda \rho_c$, where ρ_c is the closure energy density today. In order to suppress the variation of α at late times, one could also assume that $\zeta_F = 0$, or in other words, require $V(\varphi)$ and $B_F(\varphi)$ to share the same extremum. We note that most of the quintessence models analyzed in the context of changing coupling constants to date, were coupled to the electromagnetic lagrangian linearly, while the coupling to other gauge and matter fields was neglected. Continuing along the same line, it would be reasonable to allow a multitude of couplings $B_i(\varphi)$, and require $\zeta_i = 0$. This is exactly the proposal of the "least coupling" principle, introduced a decade ago by Damour and Nordtvedt [36] and Damour and Polyakov [37] in the context of the string dilaton with the primary goal to suppress the strength of the gravitational interaction mediated by φ. Here we supplement their idea by including the self-interaction potential $V(\varphi)$ with an extremum at the same value of φ as in $B_i(\varphi)$. We note that this property, a common extremum in φ, remains intact even in the presence of the radiative corrections, and therefore does not require any additional fine-tuning.

In what follows, we analyze in detail the cosmology of coupled quintessence models with

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a common extremum in $V(\varphi)$ and gauge and matter/gauge couplings $B_i(\varphi)$. We note that only a coupled quintessence model derived from Brans-Dicke theory has been analyzed to date [38, 39]. We investigate the size of the possible variation of masses and couplings in these models and find that some generic choices of $V(\varphi)$ and $B_i(\varphi)$ can be consistent with all observational requirements. This paper is organized as follows. In the next section we introduce our model and display the necessary cosmological equations. In sections 3-5, we make specific choices for the potential $V(\varphi)$ and matter/gauge couplings $B_i(\varphi)$ and find the evolution of the dark energy and dark matter energy density over the redshift. We make predictions for the variation of the coupling constants and masses and outline the choice of parameters that satisfies all existing constraints. We reach our conclusions in section 6.

2 Cosmological evolution of a scalar field in the presence of couplings to matter

We start by writing down the general equation for the interaction of a light scalar field φ with matter,

$$S_\phi = \int d^4x \sqrt{-g} \bigg\{ \frac{\bar{M}^2}{2} [\partial^\mu \phi \partial_\mu \phi - R] - V(\phi)$$

$$-\frac{B_{Fi}(\phi)}{4} F^{(i)}_{\mu\nu} F^{(i)\mu\nu} + \sum_j [\bar{\psi}_j i\not{D}\psi_j - B_j(\phi) m_j \bar{\psi}_j \psi_j] \bigg\}. \tag{2.1}$$

In this expression, ϕ is the scalar field of interest with its kinetic term normalized together with the gravitational interaction. The placement of the reduced Planck mass, $\bar{M}$, in (2.1) renders ϕ dimensionless. $B_{Fi}(\phi)$ represents the ϕ-dependence of the gauge couplings in Standard Model where the sum is over all three groups. ψ_j represents Standard Model fermions that are coupled to ϕ via the functions $B_j(\phi)$. Note that the ϕ-dependent rescaling of the matter fields and metric allows us to remove ϕ from the couplings to the kinetic terms and R. A similar rescaling in the gauge sector will lead to the appearance of a ϕ-dependence in the interaction terms between gauge fields and matter, *i.e.* precisely ϕ-dependent gauge couplings. Besides SM fermions, the sum over j includes other matter field (*i.e.* scalar Higgses, Majorana neutrinos etc.) as well as various interaction terms. Finally, any viable cosmological scenario requires cold dark matter, and we also include it in the sum with the separate coupling $B_{DM}(\phi)$. Since ϕ couples to the trace T^μ_μ of dark matter, our results are independent of the nature of the dark matter particles (scalars or fermions). Furthermore, since dark matter is the dominant component of the matter energy-density, we can simply take $B_m(\phi) \simeq B_{DM}(\phi)$.

From the above action (2.1) we can derive the equation of motion for the scalar field, ϕ:

$$\Box\phi + \frac{1}{\bar{M}^2}\frac{\partial V}{\partial \phi} = -\sum_j \frac{1}{\bar{M}^2}\frac{\partial B_j}{\partial \phi} m_j \langle \bar{\psi}_j \psi_j \rangle, \tag{2.2}$$

where $\langle \bar{\psi}_j \psi_j \rangle$ stands for the number density of a j-th fermion. Notice that at tree level, radiation does not contribute to the r.h.s. of Eq. (2.2) because its stress-energy tensor is traceless. Given a potential $V(\phi)$, the perfect fluid energy density and pressure contributions due to ϕ are:

$$\rho_\phi = \frac{1}{2}\bar{M}^2\dot{\phi}^2 + V(\phi) \tag{2.3}$$

$$p_\phi = \frac{1}{2}\bar{M}^2\dot{\phi}^2 - V(\phi) \equiv \omega_\phi \rho_\phi, \tag{2.4}$$

where the parameter ω_ϕ is related to the equation of state (EOS) of the scalar field. Combining Eqs. (2.3) and (2.4), we obtain a useful relation between the potential $V(\phi)$, the energy density of the scalar field, and ω_ϕ,

$$\rho_\phi = \frac{2V(\phi)}{(1-\omega_\phi)}. \tag{2.5}$$

The energy density ρ_ϕ and pressure p_ϕ of the scalar field contribute to the r.h.s. of Einstein's equations and yield the following Friedmann equation in a Robertson-Walker Universe,

$$H^2 = \frac{1}{3\bar{M}^2}\left(\rho_\phi + \rho_r + \rho_m\right) \equiv \frac{1}{3\bar{M}^2}\rho_{cr}, \tag{2.6}$$

where $H = \dot{a}/a$ is the Hubble expansion rate, $\bar{M}^2 = M_p^2/8\pi$ is a reduced Planck mass, ρ_r and ρ_m is the energy density of radiation and matter, and ρ_{cr} is the critical energy density. The energy density of matter and radiation contains ϕ-dependence, $i.e.$ $\rho_m = \sum_j B_j(\phi)m_j\langle\bar{\psi}_j\psi_j\rangle$. Using these definitions, we can rewrite the scalar field equation (2.2),

$$\ddot{\phi} + 3H\dot{\phi} + \frac{1}{\bar{M}^2}\frac{\partial V}{\partial \phi} = -\frac{1}{\bar{M}^2}\frac{\partial \ln B_m}{\partial \phi}\rho_m. \tag{2.7}$$

Conservation of the energy-momentum tensor gives us another equation,

$$\dot{\rho}_\phi + 3H(1+\omega_\phi)\rho_\phi = -\frac{\partial \ln B_m}{\partial \phi}\rho_m\dot{\phi}. \tag{2.8}$$

For cosmological studies that span a large range of redshifts z, it is convenient to introduce the variable x as the logarithm of the scale factor a,

$$x = \ln a = -\ln(1+z) \tag{2.9}$$

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where we choose the present scale factor $a^{(0)} = 1$. With the use of the variable x, we can rewrite the relevant system of equations for $d \ln \rho_i / dx$ in the following form,

$$\frac{d \ln \rho_m}{dx} = -3(1 + \omega_m) + \frac{\partial \ln B_m(\phi)}{\partial \phi} \frac{d\phi}{dx}, \tag{2.10}$$

$$\frac{d \ln \rho_r}{dx} = -3(1 + \omega_r), \tag{2.11}$$

$$\frac{d \ln \rho_\phi}{dx} = -3(1 + \omega_\phi) - \frac{\partial \ln B_m(\phi)}{\partial \phi} \frac{d\phi}{dx} \frac{\rho_m}{\rho_\phi}, \tag{2.12}$$

where $\omega_r = 1/3$ and $\omega_m = 0$ should be used for radiation and matter respectively.

The evolution of ϕ as a function of redshift can be found relatively easy if one is able to solve the cosmological equations and obtain ω_ϕ and $\Omega_i \equiv \rho_i / \rho_{cr}$ as a function of x. For example, the derivative of ϕ with respect to x can be obtained from Eq (2.3),

$$\left(\frac{d\phi}{dx} \right)^2 = 3\Omega_\phi(1 + \omega_\phi), \tag{2.13}$$

If ω_ϕ is close to -1, the kinetic energy of the scalar field goes to zero which occurs when the scalar field is close to the minimum of the potential.

The evolution of $\phi(x)$ expressed in terms of ω_ϕ and Ω_ϕ takes an especially simple form when $B_m(\phi)$ does not produce a significant change in the cosmological equations, and terms containing $\partial B_m(\phi)/\partial \phi$ can be neglected. In this case, the matter and radiation energy densities can be easily related, $\rho_r = (a_{eq}/a)\rho_m$, where a_{eq} is the scale factor when the radiation and matter densities are equal, and $a_{eq} \ll 1$. This allows us to rewrite Eq. (2.6) for the evolution of the scalar field energy density in a useful form,

$$\frac{d \ln(1 - \Omega_\phi)}{dx} = \Omega_\phi \left[3\omega_\phi - \left(\frac{a_{eq}}{a + a_{eq}} \right) \right]. \tag{2.14}$$

The dependence of critical density on the scale factor can be found explicitly, which allows us to express $V(\phi)$ from (2.5) as

$$V(\phi) = \frac{1}{2}(1 - \omega_\phi)\Omega_\phi \rho_{cr} = \frac{3}{2}(\bar{M}H^{(0)})^2(1 - \omega_\phi)\Omega_\phi \frac{1 - \Omega_\phi^{(0)}}{1 - \Omega_\phi} \left(\frac{a^{(0)}}{a} \right)^4 \frac{a + a_{eq}}{a^{(0)}}. \tag{2.15}$$

In this expression, $H^{(0)}$ and $1 - \Omega_\phi^{(0)} = \Omega_m^{(0)}$ are the Hubble expansion rate and the matter density today. This equation can be used further to obtain ϕ, *i.e.* by taking the logarithm of both sides of (2.15) for a simple exponential potential. The derivative of (2.15) with respect to x gives us another useful equation,

$$\frac{d \ln(1 - \omega_\phi)}{dx} = 3(1 + \omega_\phi) + \frac{\partial \ln V}{\partial \phi} \frac{d\phi}{dx}. \tag{2.16}$$

When the change in $B_m(\phi)$ is not small and cannot be neglected, all cosmological equations become considerably more complicated. The scaling of the matter energy density differs from usual a^{-3} behavior because of the changing mass, due to $B_m(\phi)$, while the scaling of radiation energy density remains unchanged,

$$\rho_m(x) = \rho_m^{(0)} a^{-3} \frac{B_m\big(\phi(x)\big)}{B_m\big(\phi(0)\big)}, \quad \text{and} \quad \rho_r(x) = \rho_r^{(0)} a^{-4} \tag{2.17}$$

$$\rho_r = \frac{a_{eq}}{a} \frac{B_m\big(\phi(x_{eq})\big)}{B_m\big(\phi(x)\big)} \rho_m \tag{2.18}$$

With the use of these relations, we generalize Eqs. (2.14) and (2.16),

$$\frac{d\ln(1-\Omega_\phi)}{dx} = \Omega_\phi\left[3\omega_\phi - \left(\frac{a_{eq}^{(c)}}{a + a_{eq}^{(c)}}\right)\right] + \left(\frac{a}{a + a_{eq}^{(c)}}\right)\frac{\partial \ln B_m}{\partial \phi}\frac{d\phi}{dx} \tag{2.19}$$

$$\frac{d\ln(1-\omega_\phi)}{dx} = 3(1+\omega_\phi) + \frac{\partial \ln V}{\partial \phi}\frac{d\phi}{dx} + \frac{(1-\Omega_\phi)}{\Omega_\phi}\left(\frac{a}{a + a_{eq}^{(c)}}\right)\frac{\partial \ln B_m}{\partial \phi}\frac{d\phi}{dx} \tag{2.20}$$

where we have introduced an auxiliary function, $a_{eq}^{(c)}(\phi) = a_{eq}B_m\big(\phi(x_{eq})\big)B_m^{-1}\big(\phi(x)\big)$.

3 Cosmological evolution of the scalar field

3.1 Relation between $V(\phi)$ and $B_i(\phi)$

Our main assumption in this work is that all functions $B_i(\phi)$ and $V(\phi)$ admit a common extremum, which is an obvious generalization of Damour-Nordtvedt and Damour-Polyakov constructions [36, 37]. Near this extremum, all functions admit an expansion

$$B_i(\phi) = 1 + \frac{1}{2}\xi_i\phi^2 + ...; \qquad V(\phi) = V_0(1 + \frac{1}{2}\lambda\phi^2 + ...), \tag{3.1}$$

where ξ_i and λ are dimensionless parameters, while V_0 is of the order of the dark energy density today.

In order to be able to start our analysis, we must specify the potential $V(\phi)$ and functions $B_i(\phi)$. Without any serious guidance from the underlying theory, their choice is completely *ad hoc*, and the only constraints are those coming from observations. A large number of quintessence potentials $V(\phi)$ has been analyzed in the past, but the parameter space of functions $B_m(\phi)$ and $B_F(\phi)$ remains relatively unexplored. To this end, we propose an ansatz that would relate $B_i(\phi)$ and $V(\phi)$. Since we are going to consider only positive-definite functions, we choose a two-parameter relation,

$$B_i(\phi) = \left(\frac{b_i + V(\phi)/V_0}{1 + b_i}\right)^{n_i}, \quad \text{where} \ \ b_i + 1 > 0 \tag{3.2}$$

This relation is sufficiently generic, and varying the two dimensionless parameters n_i and b_i for each function $B_i(\phi)$ allows us to cover a wide range of possibilities. The expansion of (3.2) near the extremum allows us to relate ξ_i and λ,

$$\xi_i = \frac{\lambda n_i}{1 + b_i}. \tag{3.3}$$

If the quintessence field changes by $\Delta\phi \sim O(1)$ between redshifts $z \sim 1$ and $z = 0$, the constraints on the variation of masses and couplings would normally require small ξ_i which may be due to small n_i and/or a large value for b_i. In a typical tracker regime, $V(\phi)$ changes by many orders of magnitude, and at early cosmological epochs, the value of $V(\phi)$ can be much larger than the normalization scale b_i. In this case, $B_i(\phi) \sim (V(\phi))^n$.

In the next two sections, we analyze two types of potentials, which in the asymptotic regime of tracking behavior, scale as $\exp(\phi^2)$ and $\exp(\phi)$. We first assume that the couplings of ϕ to all matter fields are rather small and have no backreaction on the cosmological evolution of ϕ. We then allow for a substantial coupling $B_m(\phi)$ to dark matter and show the impact of that on the dynamics of the quintessence field. By comparing the evolution of $B_F(\phi)$ for electromagnetic field to observational constraints, we can determine the allowed range of parameters for n_F and b_F.

3.2   Quintessence with $\exp(\lambda\phi^2/2)$ and $\cosh(\lambda\phi)$ type potentials

We next consider two sample potentials which can combine the properties of a tracking solution with a minimum that provides the dark energy at late times:

$$\text{case A}: \qquad V(\phi) \;=\; V_0 \exp\Big(\frac{\lambda\phi^2}{2}\Big) \tag{3.4}$$

$$\text{case B}: \qquad V(\phi) \;=\; V_0 \cosh(\lambda\phi) \tag{3.5}$$

Both potentials have the same expansion near the minimum (3.1) and are sufficiently easy to analyze. For now, we set $B_F = B_m = 1$. Closely related potentials have been expansively analyzed in the past by several groups [7, 9].

Case A.

While the evolution equation for Ω_ϕ is unchanged from Eq. (2.14), the evolution equation for ω_ϕ can be rewritten as

$$\frac{d\ln(1 - \omega_\phi)}{dx} = 3(1 + \omega_\phi) \pm \lambda\phi\sqrt{3\Omega_\phi(1 + \omega_\phi)}, \tag{3.6}$$

where the choice of plus(minus) depends on whether the evolution of ϕ starts from negative(positive) values. The dependence of ϕ can be obtained directly from (2.15),

$$\frac{\lambda}{2}\phi^2 = -4x + \ln(a + a_{eq}) + \ln\left(\frac{\Omega_\phi(1 - \omega_\phi)}{1 - \Omega_\phi}\right) + \ln\left[\frac{3(H^{(0)}\bar{M})^2(1 - \Omega_\phi^{(0)})}{2V_0}\right] \tag{3.7}$$

All terms in Eq. (3.7) other than the $(-4x)$-term do not change significantly over the history of Universe. This is similar to the evolution of a scalar field in the tracker regime for an exponential potential, where $\ln V(\phi) \sim \text{const} - 3(1 + \omega_i)x$ [11].

An advantage of a tracker-type potential is that the cosmological evolution of the scalar field ϕ during the observationally relevant epoch, *i.e.* $z \lesssim 10^{10}$ is insensitive to the choice of initial conditions for ϕ that can be specified deep inside the radiation-dominated epoch. In order to exhibit tracking behavior, the potential must satisfy two conditions [4]:

$$\Gamma \equiv \frac{V''V}{(V')^2} \geq 1 \quad \text{and} \quad \left| \frac{d\ln(\Gamma - 1)}{dx} \right| \ll 1. \tag{3.8}$$

The simple exponential potential provides a tracker solution with $\Gamma = 1$.

For the potential (3.4), we find

$$\Gamma = 1 + \frac{1}{\lambda\phi^2}, \tag{3.9}$$

which is larger than 1 for any positive λ. For the second condition we obtain

$$\left| \frac{d\ln(\Gamma - 1)}{dx} \right| = \frac{2\sqrt{3\Omega_\phi(1 + \omega_\phi)}}{|\phi|}, \tag{3.10}$$

which is much smaller than one as long as ϕ is far from the minimum, i.e. at all cosmological epochs except for late times.

To be compatible with observational data, quintessence models must satisfy the following conditions: 1) The energy density of quintessence must be subdominant during Big Bang nucleosynthesis (BBN) [3], $\Omega_\phi^{(BBN)}(x \sim -23) < 0.2$ at $T \sim 1$ MeV; 2) the energy density at present must be compatible with the preferred range for dark energy, $0.60 \leq \Omega_\phi^{(0)}(x = 0) \leq 0.85$ [40]. The evolution of energy density components, Ω_r, Ω_m, and Ω_ϕ is shown in Fig. 1a along with the dark energy equation of state. In the very early universe, the EOS parameter has an oscillatory behavior as does the density parameter Ω_ϕ which quickly converges to an attractor solution washing away any memory of initial conditions. As one can see, the constraint of $\Omega_\phi < 0.2$ from nucleosynthesis is easily satisfied in this model. We also note that during the radiation dominated era, $\omega_\phi \approx \omega_r = 1/3$ as in other tracker solutions. The transition to a matter dominated Universe occurs around $x = -8.6$ ($z \sim 5400$). During the matter dominated stage, ω_ϕ begins to drop sharply towards its final value of $\omega_\phi = -1$. At $x = -0.5$ ($z \simeq 0.6$), the Universe begins to be dominated by the quintessence field. In Fig. 1b, we show the present value of ω_ϕ as a function of the coupling, λ, for three different values of the present value of Ω_ϕ. For large values of λ, the potential is relatively steep and the field quickly evolves towards its minimum where $V \simeq V_0$, which must be tuned close to the present cosmological constant value. In this case,
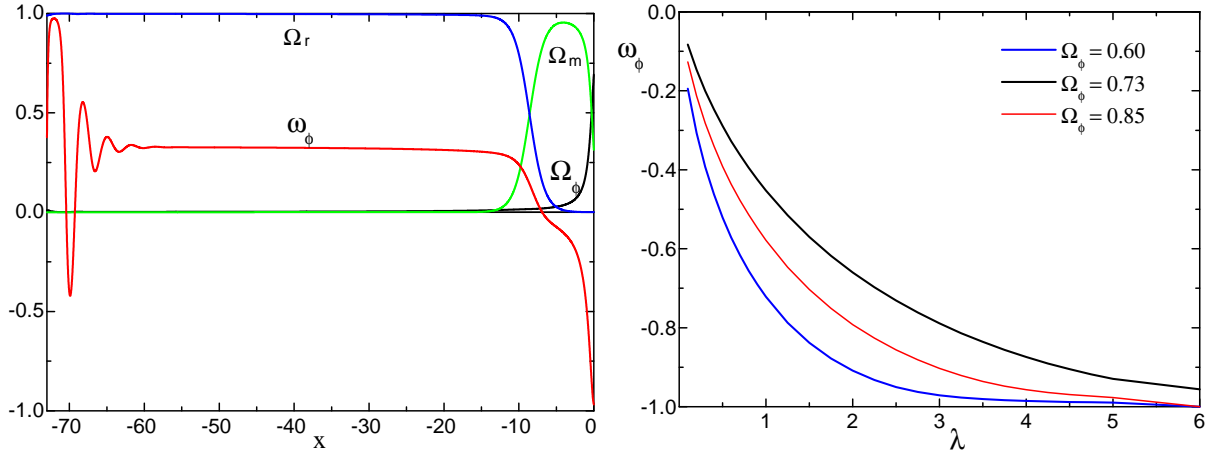
Figure 1: The $\exp(\lambda\phi^2/2)$ potential. a) The cosmological evolution of the equation of state parameter, ω_ϕ, and the energy density parameters, Ω_i, of each component for $\lambda = 5$. The evolution of the parameters is similar for other choices of λ. b) The dark energy equation of state parameter ω_ϕ as a function of λ for an acceptable range of the dark energy density $(0.6 \leq \Omega_\phi^{(0)} \leq 0.85)$.

$\omega_\phi \to -1$. In contrast, for small values of λ, the potential is relatively shallow, and ϕ is still evolving. As one can see, applying the constraint [41], $\omega_\phi^{(0)} < -0.6$ gives the limit $\lambda \gtrsim 1.5$ (for the case of $\Omega_\phi = 0.73$). When the present value of ϕ is not at the minimum ($\phi = 0$), the constant, V_0 must be adjusted to obtain the desired value of Ω_ϕ today.

Case B.

As one might expect, the potential (3.5) gives a very similar result as one would obtain for a simple exponential potential ($V(\phi) \propto \exp(-\lambda\phi)$) at early times. Therefore, for $\lambda\phi \gtrsim 1$, *i.e.* far from the minimum, the scalar field is simply

$$\lambda\phi \simeq -4x + \ln(a + a_{eq}) + \ln\left(\frac{\Omega_\phi(1 - \omega_\phi)}{1 - \Omega_\phi}\right) + \ln\left[\frac{3(H^{(0)}\bar{M})^2(1 - \Omega_\phi^{(0)})}{2V_0}\right]. \qquad (3.11)$$

For this potential, the tracking parameter Γ is,

$$\Gamma = 1 + \frac{1}{\sinh^2(\lambda\phi)}. \qquad (3.12)$$

Clearly, at late times, as ϕ approaches 0, the tracking condition will be violated which opens up the possibility that the Universe will be dominated by the scalar field, in contrast to the simple exponential potential. A comparison of the late time behavior between the exponential and cosh type potentials is shown in Fig.2.
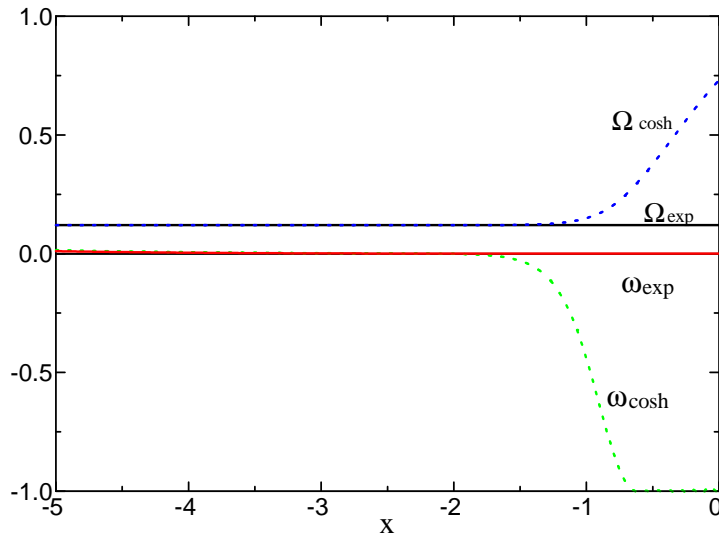
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Figure 2: The evolution of Ω_ϕ and ω_ϕ for the simple exponential potential (solid) and the cosh potential (dotted). The main difference between two potentials appears at late times. We use $\lambda = 5$ for both potentials.

In Fig. 3a, we show the full evolution of Ω_r, Ω_m, Ω_ϕ, and ω_ϕ. In this case, matter domination occurs at $x = -8.2$ ($z \simeq 3500$), and quintessence domination occurs at $x = -0.3$ ($z \simeq 0.35$). The cosh-type potential turns out to be more restricted by observational requirements than the potential (3.4). In Fig. 3b, we show the value of $\Omega_\phi(x = -23)$ corresponding to its BBN value as a function of the coupling λ. We see that the constraint $\Omega_\phi < 0.2$ implies $4 \leq \lambda$.

4   Time variation of the fine structure constant

To study the cosmological evolution of the fine structure constant we use the following relation between B_F and α,

$$\frac{\Delta\alpha(z)}{\alpha} \equiv \frac{\alpha(z) - \alpha(0)}{\alpha(0)} = \frac{B_F(\phi(0))}{B_F(\phi(z))} - 1. \tag{4.1}$$

Note that the present value of the field $\phi(0)$ is close to zero.

We assume that the backreaction of $B_i(\phi)$ on the cosmological evolution of ϕ itself is small, and therefore can use the results of the previous section. All physical processes
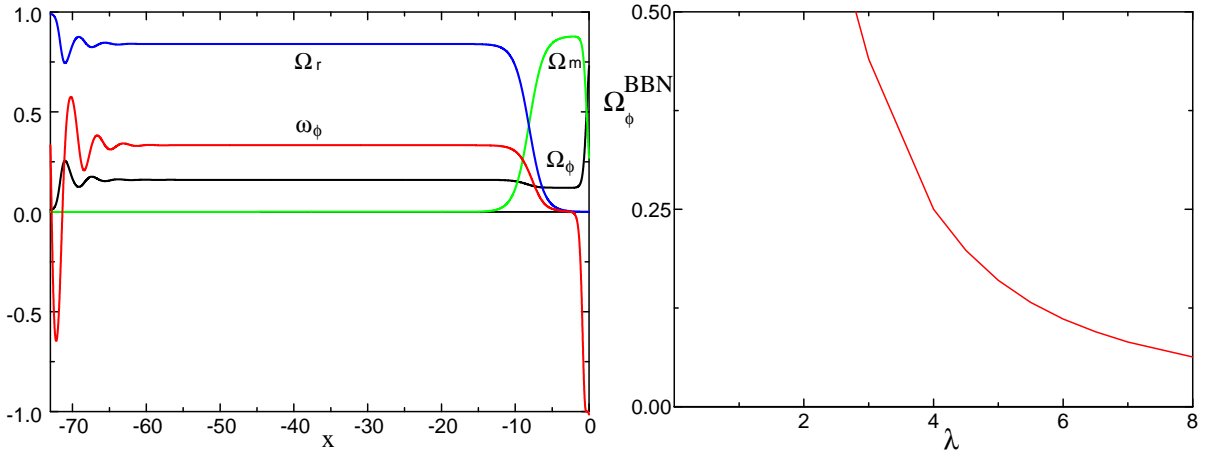
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Figure 3: The $\cosh(\lambda\phi)$ potential. a) The cosmological evolution of the equation of state parameter, ω_ϕ, and the energy density parameters, Ω_i, of each component for $\lambda = 5$. The evolution of the parameters is similar for other choices of λ. b) The quintessence density parameter Ω_ϕ evaluated at the time of BBN as a function of λ.

sensitive to changes in α can be separated into two broad categories: those that correspond to relatively low redshifts, $z \sim 1$ and smaller, and high-redshift phenomena such as BBN and the cosmic microwave background anisotropies.

4.1 Late time evolution

For the late time evolution of α we can use the expansion of $B_F(\phi)$ given in (3.1). In this case, the expression for $\Delta(\alpha)$ takes the following simple form,

$$\frac{\Delta\alpha(z)}{\alpha} = \frac{\xi}{2}\left(\phi^2(0) - \phi^2(z)\right), \tag{4.2}$$

where we dropped the subscript F in ξ_F to be concise. For the exponential of ϕ^2 potential (3.4), this expression can be found in a more explicit form with the use of (3.7),

$$\frac{\Delta\alpha(x)}{\alpha} = \frac{\xi}{\lambda}\left[-3x + \ln\left(\frac{\Omega_\phi(1-\Omega_\phi^{(0)})(1-\omega_\phi)}{\Omega_\phi^{(0)}(1-\Omega_\phi)(1-\omega_\phi^{(0)})}\right)\right]. \tag{4.3}$$

Using the result of the previous section, we can predict the evolution of α over redshift in terms of two parameters, ξ and λ. We choose two characteristic values of ξ, based on two QSO results. To be consistent with the non-zero result for $\Delta\alpha$ by Murphy et al. [21], we choose ξ in such a way that $\Delta\alpha/\alpha = -5.4 \times 10^{-6}$ at a redshift of 3. Another option that we explore is $|\Delta\alpha/\alpha| \leq 6 \times 10^{-7}$ at $z = 1.5$, which is motivated by the experimental accuracy of Chand et al. [22]. For definiteness, in the second case we choose $\Delta\alpha/\alpha = -7 \times 10^{-6}$.

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The results for the choice $\Delta\alpha/\alpha = -5.4 \times 10^{-6}$ are plotted in Fig. 4. The family of solid curves corresponds to different choices of the coupling $\lambda = 1, 4$, and 5 in (3.4). It is easy to see that the evolution of α is approximately linear over a large range of redshifts for $z > 1$. This follows directly from (4.3) where only the $-3x$ term is changing at these redshifts. In panel b) of Fig. 4 we show a blow-up of panel a) for $z \leq 0.5$. This will facilitate the comparison between this model and the experimental constraints at low z. At late times (at low z), the evolution of α slows down considerably. For comparison, we present similar results for the case of the linear coupling ζ between ϕ and $F_{\mu\nu}F^{\mu\nu}$, for the same choice of initial conditions. The corresponding curves are plotted in Fig. 4 by dashed-dotted lines. The main qualitative difference between these two choices, ξ or ζ is at late times. When $\zeta = 0$ and $\xi \neq 0$, the fine structure constant evolves very slowly, whereas for $\zeta \neq 0$ the evolution of α remains approximately linear.

In Fig. 5, we show the analogous results for the evolution of α for the potential (3.5). Once again, we show results for three choices of λ; in this case, we have used $\lambda = 4, 4.5$, and 5. Notice that in this case, α overshoots its present day value. This is due to the fact that with these choices of λ, ϕ rolls past the minimum (at $\phi = 0$) and begins an oscillatory behavior as it settles to the minimum of the potential. For the quadratic coupling, variations in α do not change sign thus causing $\Delta\alpha$ to reach the value 0, before becoming negative again. Damped oscillations of ϕ (and hence α) will continue to settle towards the minimum for which $\Delta\alpha = 0$.

We now turn to the question of how the sensitivity to $\Delta\alpha/\alpha$ achieved in Refs. [21, 22] compares to other probes of $\Delta\alpha/\alpha$ at smaller redshifts. In particular, we calculate $\Delta\alpha/\alpha$ at $z = 0.14$ relevant for the Oklo bound [25], the time average $\overline{\Delta\alpha}/\alpha$ between the redshifts 0 and $z = 0.45$ which is constrained by meteoritic data [26, 27], and the rate of change in α today, $\dot{\alpha}/\alpha$ [45]. In addition to the observables directly related to the fine structure constant, we also calculate the differential acceleration of two bodies towards a common attractor, limited by the precision tests of the universality of the gravitational force [35, 46]. To calculate $\overline{\Delta g}/g$ we use the calculation of Ref. [35], and conservatively assume that the coupling of the scalar field to nuclei is mediated only by the electromagnetic interactions. The calculations of Ref. [35] are done for the linear coupling ζ, but the generalization to the case of $\zeta = 0, \xi \neq 0$ is straightforward, via the substitution $\zeta \to \xi\phi(z = 0)$ [37]. The results of these calculations are compiled in Table 1.

In Table 1, we display specific results for both forms of the potential, $V(\phi)$, considered; both with $\lambda = 5$. We also show results for both sets of normalization [21, 22] and for
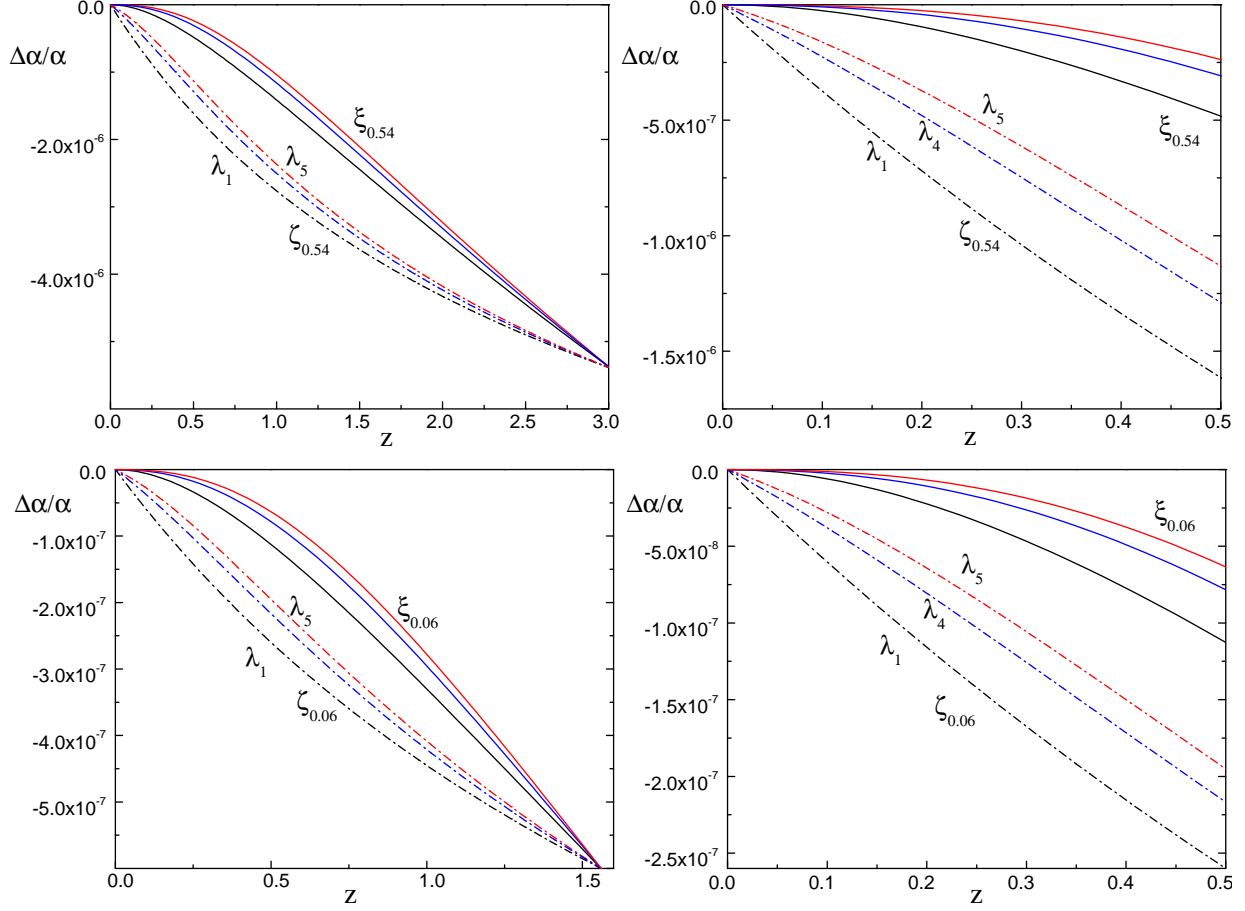
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Figure 4: a. The evolution of $\Delta\alpha/\alpha$ over the redshift range $0 \leq z \leq 3$ driven by the potential (3.4). Panels a) and b) use the common normalization $\Delta\alpha/\alpha = -0.54 \times 10^{-5}$ at $z = 3$. Figures c) and d) use the common normalization $\Delta\alpha/\alpha = -0.06 \times 10^{-5}$ at $z = 1.5$. The solid lines correspond to the choice $\zeta = 0$ and $\xi \neq 0$, whereas the dashed-dotted lines allow $\zeta \neq 0$.
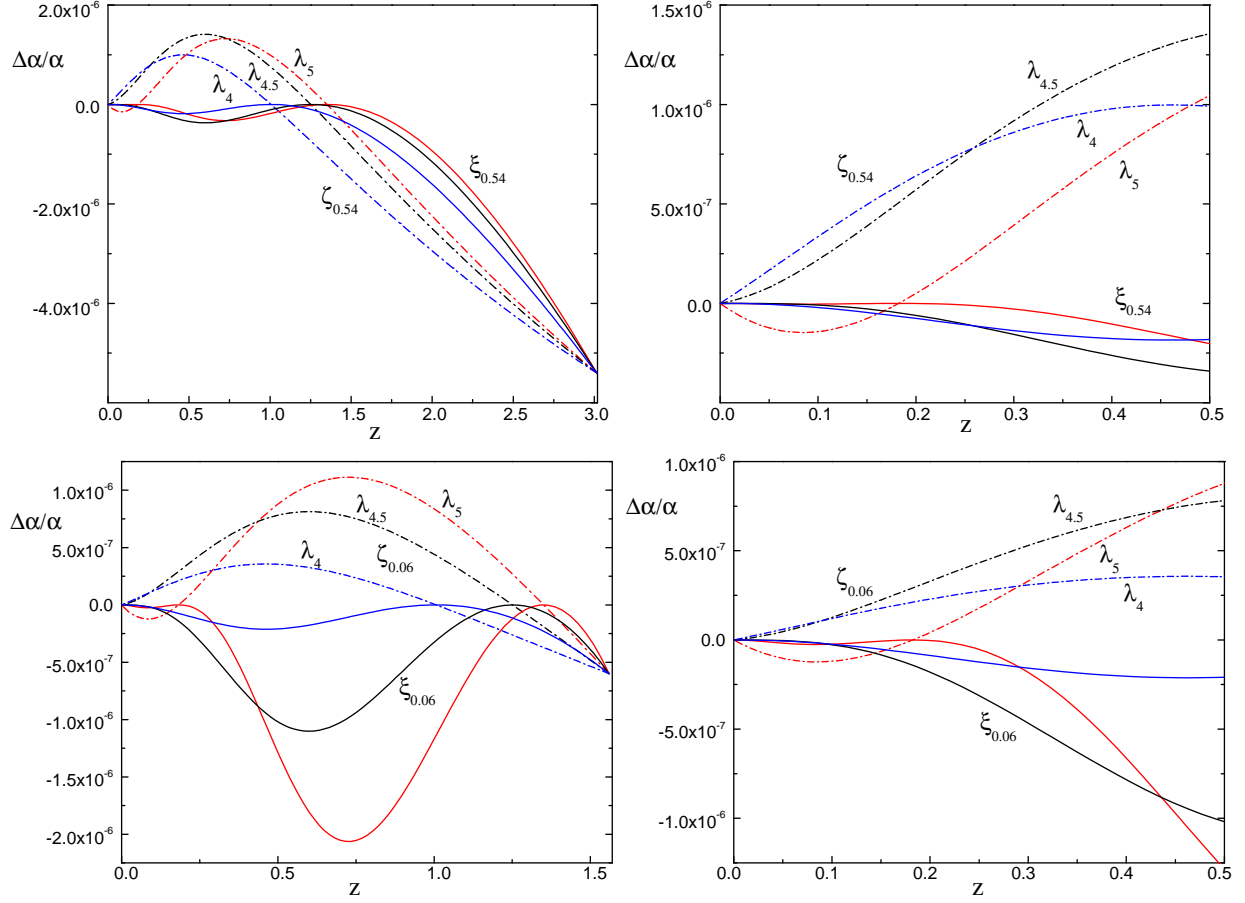
Figure 5: As in Fig. 4 for the potential 3.5. Here $\lambda = 4, 4.5,$ and 5.

Table 1: Results for $\Delta\alpha/\alpha$ for comparisons with the Oklo, meteoritic, present-day, and equivalence principle bounds. Values of $\Delta\alpha/\alpha$, ξ, and ζ have been scaled by a factor of 10^6, values of $\dot\alpha/\alpha$ and $\Delta g/g$ have been scaled by a factor of 10^{17}.

$\frac{V(\phi)}{V_0}$	$(\frac{\Delta\alpha}{\alpha})_3$	$(\frac{\Delta\alpha}{\alpha})_{1.5}$	ξ	ζ	$(\frac{\Delta\alpha}{\alpha})_{0.14}$	$(\frac{\Delta\alpha}{\alpha})_{0.45}$	$\frac{\dot\alpha}{\alpha}$ (yr^{-1})	$\frac{\Delta g}{g}$
$\exp(\lambda\phi^2/2)$	-5.4	-3.4	0	-7.6	-0.24	-0.42	9.6	-2.5
	-5.4	-2.1	22	0	-0.011	-0.05	0.048	-0.074
	-0.93	-0.6	0	-1.3	-0.041	-0.073	1.7	-0.076
	-1.4	-0.6	5.8	0	-0.0028	-0.013	0.013	-0.0053
$\cosh(\lambda\phi)$	-5.4	-0.54	0	-29	-0.097	0.20	0.17	-36
	-5.4	-0.054	310	0	-0.0017	-0.029	0.0014	-1.5
	-4.5	-0.6	0	-24	-0.082	0.17	0.16	-26
	-34	-0.6	1900	0	-0.011	-0.18	0.012	-59

both a linear ($\zeta \neq 0$) and quadratic ($\xi \neq 0$) coupling of ϕ to F^2. Here, we will not consider the constraints on α when relations between all three gauge couplings are assumed [42, 43, 44]. The latter are typically a factor of 100 times stronger. The Oklo bound [25] is roughly $\Delta\alpha/\alpha \lesssim 10^{-7}$ Therefore any result in Table 1 with a value of $(\frac{\Delta\alpha}{\alpha})_{0.14}$ in excess of 0.1 is excluded. For the $\exp(\lambda\phi^2/2)$ potential, the linear coupling with the -0.54×10^{-5} normalization is excluded, while the other cases are all allowed. From Fig. 4, we see also that smaller values of λ lead to larger variations in α at $z = 0.14$. For the $\cosh(\lambda\phi)$ potential, all cases with $\lambda = 5$ are allowed. However, as one can see from Fig. 5, even the slightly lower values of λ shown lead to excessive variations in α for the linear coupling.

A closer look at Table 1 reveals an interesting observation. A non-zero value for $\Delta\alpha/\alpha$ at the level of $\sim 0.5 \times 10^{-5}$ at $z = 3$ can be perfectly consistent with the bound $|\Delta\alpha/\alpha(z = 1.5)| \leq 0.6 \times 10^{-6}$ because of the highly non-linear evolution of the scalar field. Indeed, the quadratic coupling for the cosh-like potential shows that the ratio $\Delta\alpha(z = 1.5)/\Delta\alpha(z = 3)$ can be as small as 0.01. This implies that in certain models the non-zero result of Ref. [21] for $\Delta\alpha/\alpha$ that spans a rather large range of redshifts and the better-sensitivity zero-result measurement of $\Delta\alpha/\alpha$ at one redshift $z = 1.5$ may not necessarily be contradictory. This would require, however, that the dominant source of the effect observed in [21] is due to their high redshift absorbers ($z > 1.8$) which also carry the largest uncertainties. Nevertheless, we see that even in the simple models considered here, the change of α in time can be compatible with both claims without any special fine-tuning.

The meteoritic bound was discussed in detail in [26], where the bound $-8 \times 10^{-7} < \Delta\alpha/\alpha < 24 \times 10^{-7}$ was derived. Note that in this case, the bound relates to the time average of $\Delta\alpha/\alpha$ which is the quantity given in Table 1. Thus values in the table should

be restricted between -0.8 and 2.4. All models shown satisfy this constraint.

The combined result of several recent atomic clock measurements limit the present day rate of change in the fine-structure constant to $\dot{\alpha}/\alpha = (-0.9 \pm 4.2) \times 10^{-15}$ [45]. All models considered easily satisfy this bound. Finally, we compute the differential acceleration of the Earth-Moon system towards the Sun. The limit on $\Delta g/g$ is 0.9×10^{-12} [46]. As one can see, all of the models considered easily satisfy this bound as well.

4.2 Early time evolution

The experimental searches for variations in α provide a very useful constraint (or determination, according to Ref. [21]) on the size of the quadratic term, $\xi = \lambda n/(1 + b)$, where we dropped the subscripts in n_F and b_F. Since ξ is typically 10^{-4} or smaller, one cannot determine whether this the result should be interpreted as $b \gg 1$ or $n \ll 1$ or both.

However, there are two additional important observables, $\Delta\alpha$ at the cosmic microwave background epoch [47], and at BBN [42, 44, 48] that will provide more information on b and n. In fact, since the evolution of α at early times is monotonic, and since BBN and CMB data provide a similar level of sensitivity of order a few per cent to $\Delta\alpha/\alpha$, we will just use the BBN constraint as it extends much farther back in time.

For both potentials, (3.4) and (3.5), we calculate the corresponding values of the field at the BBN redshift, $z \simeq 10^{10}$.

$$\phi_{BBN} \simeq -5.52 \ \text{ for } \ V(\phi) = V_0 \exp(\lambda\phi^2/2) \tag{4.4}$$

$$\phi_{BBN} \simeq -11.9 \ \text{ for } \ V(\phi) = V_0 \cosh(\lambda\phi) \tag{4.5}$$

These values of ϕ_{BBN} are calculated for the choice of $\lambda = 5$, and can be used to estimate the size of the correction to the fine structure constant, $\Delta\alpha/\alpha_{BBN}$. Alternatively, we can use the scaling of the $V(\phi)$ with a in Eq. (2.15) to get the same result. We conservatively assume that this quantity is limited at the level of 0.06 (see *e.g.* for the latest re-analysis of this bound [49]) based on the new derived ^{4}He abundance of $Y_p = 0.249 \pm 0.009$ [50]. The connection between V and B_F that we impose (3.2) does not allow for a linearized form of $B_F(\phi)$ (3.1) at the high redshift associated with BBN, because the potentials we consider in this paper are very steep.

Using Eq. (4.1) together with the ansatz (3.2), the constraint $\Delta\alpha/\alpha(\phi_{BBN})$ becomes

$$\left(\frac{1 + b}{b + V(\phi_{BBN})/V_0}\right)^n > 0.94 \tag{4.6}$$

This constraint is shown in Fig. 6 for our two choices of potentials, both with $\lambda = 5$. In the light shaded region, both potentials satisfy the BBN bound on α. In the medium shaded
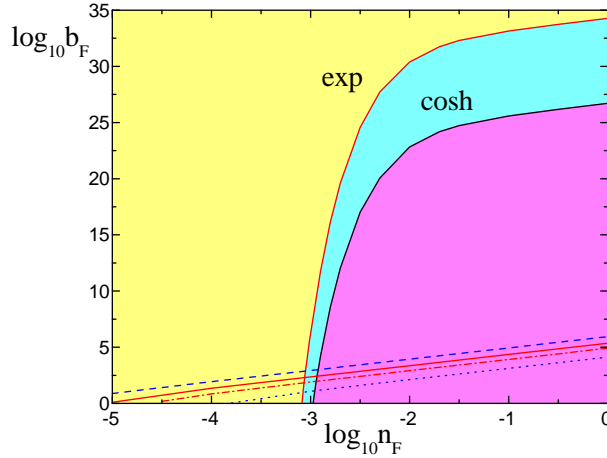
Figure 6: Constraints on the parametrized relation between B and V coming from the early- and late-time evolution of the scalar field. The constraints from the early-time evolution determine the allowed region in the $b - n$ plane for the both potentials (light shaded) and for the $\cosh(\phi)$ potential alone (medium shaded). Both potentials fail the constraint in the dark shaded region. The early-time evolution constraint is shown by the near horizontal lines at low b. The values for ξ are taken from the table: $\xi \times 10^6 = 5.8, 22, 310$, and 1900 are shown by the dashed, solid, dot-dashed, and dotted lines respectively.

region, only the $\cosh(\phi)$ potential satisfies the bound. In the dark shaded region, both potentials lead to variations in α at the time of BBN which are in excess of the constraint. Also shown in Fig. 6 is the constraint obtained in the previous section from the late-time evolution of ϕ. Here we show the relation $\xi = \lambda n/(1 + b)$ for the four values of ξ given in Table 1. The values of $\xi \times 10^6 = 5.8, 22, 310$, and 1900 are shown by the dashed, solid, dot-dashed, and dotted lines respectively. As one can see, when $n \lesssim 10^{-3}$, the BBN constraint effectively disappears. At the same time, the choice of $\xi \sim 10^{-4}$ and $n > 10^{-3}$ is clearly forbidden by the BBN constraint. Given this constraint on n, we can see that only very shallow exponential profiles of $B_F(\phi)$ are allowed.

5   Coupled quintessence

In this section, we investigate changes to the cosmological evolution of the scalar field that can be induced by a non-trivial function $B_m(\phi)$. The case of the so-called "coupled quintessence" has been addressed in the literature already [38], but was specialized to the particular case of an exponential potential with a linear coupling to matter.

The function $B_m(\phi)$ expresses the ϕ dependence of the mass of any non-relativistic

particle, including dark matter particles. It is clear that if this mass experiences significant changes during the evolution of the Universe, the scaling of the matter energy density will differ from a^{-3}, while the number density of dark matter particles n will continue to scale as a^{-3}. The cosmological evolution of ϕ also changes, because ϕ is driven by an effective potential that now consists of two parts,

$$V(\phi)_{\text{eff}} = V(\phi) + \rho_m(\phi) = V(\phi) + B_m(\phi)n. \tag{5.1}$$

It is not possible to say *a priori* which term is more important for the cosmological evolution of ϕ.

If $B_m(\phi)$ coincides with $B_F(\phi)$, we can use the insight gained from the previous section to conclude that $B_m(\phi)$ would provide no significant change on the cosmology from $z = 0$ to z_{CMB}. This is because the $\Delta\alpha/\alpha$ constraints can be re-interpreted as a very tight bounds on n_F and b_F, and as a consequence on n_m and b_m. However, $B_m(\phi)$ and $B_F(\phi)$ need not be the same. In fact, in simplest models (for example in the Brans-Dicke model, or string-inspired models with a modulus field ϕ), these functions typically are different. In the rest of this section, we assume that $B_m(\phi)$ and $B_F(\phi)$ and not related and investigate possible constraints on n_m and b_m.

For definiteness we choose $b_m = 1$, and then $B_m(\phi) = (V(\phi))^{n_m}$, according to our parametrization. If we choose $n_m > 0$, the effect of $B_m(\phi)$ causes ρ_m scale faster than a^{-3}. In fact, if the dynamics of the scalar field is driven mostly by $V(\phi)$ in the radiation dominated epoch, we can derive a simple upper bound on n_m. Indeed, according to (2.15), the scaling of $V(\phi)$ in the radiation-dominated regime is a^{-4}. Then it is clear that $B_m(\phi) \sim V(\phi)^{1/4}$ would lead to B_m scaling as a^{-1}, which also means that $\rho_m \sim a^{-4}$. Thus, $n_m < 1/4$, since for $n_m \geq 1/4$ the dark matter energy density would be redshifted in the same way or even *faster* than radiation. This would for example, greatly increase the energy density of dark matter during BBN. Therefore, $n_m \leq 0.25$. In fact if we argue that the energy density in matter should not contribute more than the energy density in radiation at BBN, we can obtain a slightly stronger bound. For simplicity, let us take the ratio of matter to radiation, r, today to be 10^4 (a more accurate number would be ~ 5500). Then at the time of BBN, the ratio would be

$$r_{BBN} \sim r_0 a_{BBN}^{(1-4n)} \tag{5.2}$$

For $r_{BBN} \lesssim 1$, $r_0 \sim 10^4$, and $a_{BBN} \sim 10^{-10}$, we have $n \lesssim 6/40$.

In Fig. 7 we show the changes in the cosmological evolution of the scalar field, matter and radiation that result from choosing $n_m = 0.1$. As one can see, there are several important differences from the uncoupled case. Because of the change in the mass of the dark matter (as described above), matter domination occurs somewhat earlier. In this case, matter domination starts at $x \simeq -9.6$ or at $z \simeq 15000$ and ends at $x \simeq -0.33$ corresponding to $z \simeq 0.4$. Perhaps the most dramatic difference is seen in the evolution of the EOS parameter
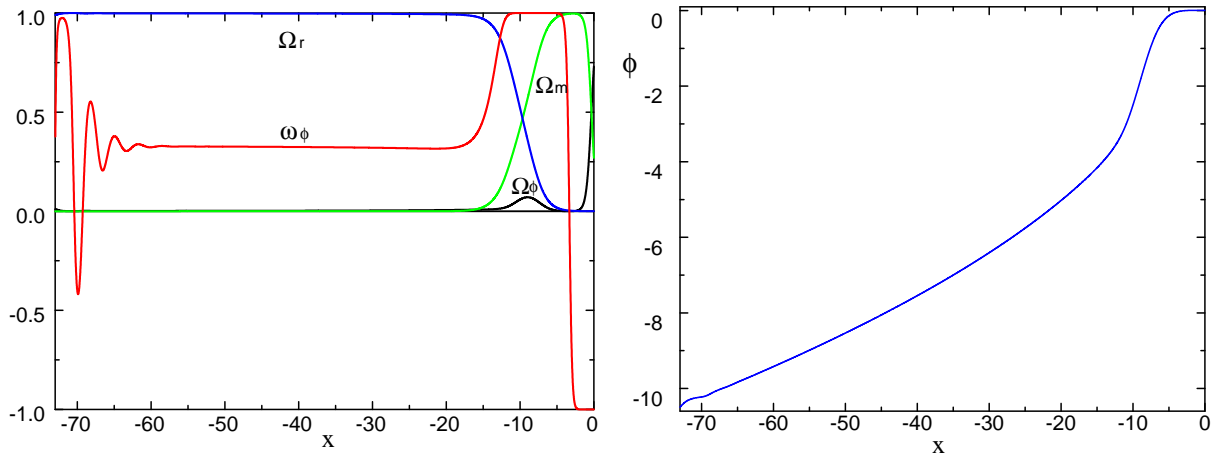
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Figure 7: a) The evolution of different components of the energy density as a function of x for the case of $n_m = 0.1$ and $b_m = 1$, b) The evolution of ϕ under the same conditions.

ω_ϕ. Instead of falling from the tracking value of $1/3$ towards -1, as in the uncoupled case, here we see that ω_ϕ increases towards $+1$, before ultimately dropping to -1 at later times. This is due to the fact that when the matter density increases, the effect on the potential $V(\phi)_{\text{eff}}$ given in Eq. (5.1) drives the evolution of ϕ faster. The field begins to quickly roll and the energy density in ϕ becomes dominated by kinetic rather than potential energy. The evolution of ϕ is shown in Fig. 7b. As one can see, around $x \sim -10$, ϕ begins to change rapidly. Hence $\omega_\phi \approx 1$. (This is also the reason for the curious bump in Ω_ϕ seen in Fig. 7a.) However, ultimately ϕ approaches and settles at the minimum of the potential. At that time, ω_ϕ quickly moves towards -1 and the Universe becomes dominated by the dark energy.

Finally we comment on the effect of the coupling to matter on the variation of α. The evolution of $\Delta\alpha$ is shown in Fig. 8 using the potential (3.4) with $\lambda = 5$. One should note that because of the matter coupling, the field ϕ has moved quickly and at the present time has already passed the minimum twice for our choice of parameters. At late times ($z < 3$), the field is significantly closer to the minimum relative to the uncoupled case. As a result, the normalization using the QSO measurements results in significantly larger couplings, ζ and/or ξ. As one can see from the figure and Table 2, despite the fact that the field is closer to the minimum, the constraints from Oklo and the meteoritic abundances are in fact more severe. Only the results based on the normalization using $\Delta\alpha/\alpha = 0.06 \times 10^{-5}$ are compatible with these constraints. Similarly the differential acceleration is predicted to be much larger in this case, but the equivalence principle constraints are still satisfied.

In contrast, because the field has nearly settled to its minimum, $\dot{\phi}$ and hence $\dot{\alpha}$ is much smaller than in the uncoupled case.

One can also determine the bounds on n_F and b_F for this model. The late-time evolution constraint is very similar to, but slightly weaker than the result shown in Fig. 6 for the $\exp(\lambda\phi^2/2)$ case. The early-time evolution constraint is weaker than all of the cases shown in Fig. 6 and does not exclude any region of the $b - n$ plane which is not already excluded by the late-time constraint.

Table 2: As in Table 1 for the coupled case with $B_m \neq 0$, and $b_m = 1$ and $n_m = 0.1$.

$\frac{V(\phi)}{V_0}$	$\left(\frac{\Delta\alpha}{\alpha}\right)_3$	$\left(\frac{\Delta\alpha}{\alpha}\right)_{1.5}$	ξ	ζ	$\left(\frac{\Delta\alpha}{\alpha}\right)_{0.14}$	$\left(\frac{\overline{\Delta\alpha}}{\alpha}\right)_{0.45}$	$\frac{\dot{\alpha}}{\alpha}$ (yr^{-1})	$\frac{\Delta g}{g}$
$\exp(\lambda\phi^2/2)$	-5.4	-5.7	0	-150	-0.85	-1.3	0.38	-1000
	-5.4	-7.4	40000	0	2.5	2.6	-0.034	-13000
	-0.56	-0.6	0	-16	-0.089	-0.14	0.039	-11
	-0.44	-0.6	3200	0	0.20	0.21	-0.0027	-88

6   Conclusions

We have analyzed the cosmological evolution of the scalar field driven by its self-interaction potential, $V(\phi)$, and its possible couplings to matter, $B_m(\phi)$. Following an original Damour-Polyakov hypothesis, we assumed a common extremum for the functions $V(\phi)$ and $B_m(\phi)$ so that the evolution of the scalar field is driven towards that extremum, chosen here to be at $\phi = 0$. With our choices of potentials, the evolution of the scalar field occurred in the tracking regime throughout the radiation dominated epoch. For definiteness we chose two potentials that ensure tracking and have a minimum, $V(\phi) = V_0 \exp(\lambda\phi^2/2)$ and $V(\phi) = \cosh(\lambda\phi)$.

We have seen that the coupling of the scalar field to the electromagnetic field is severely restricted by the late-time evolution of the field due to limits obtained from spectral measurements of quasar absorption systems, the Oklo phenomenon and meteoritic data. Models with quadratic couplings of the scalar field to radiation are typically more sensitive to the quasar absorption spectra- derived constraints than to the Oklo bound. The late-time evolution of the scalar field can be very nonlinear, and $\Delta\alpha/\alpha$ can naturally be close to zero at certain redshifts, while having significant deviations over a range of the redshifts. Thus the experimental results, Refs. [21] and [22], claiming a non-zero result over a range of redshifts
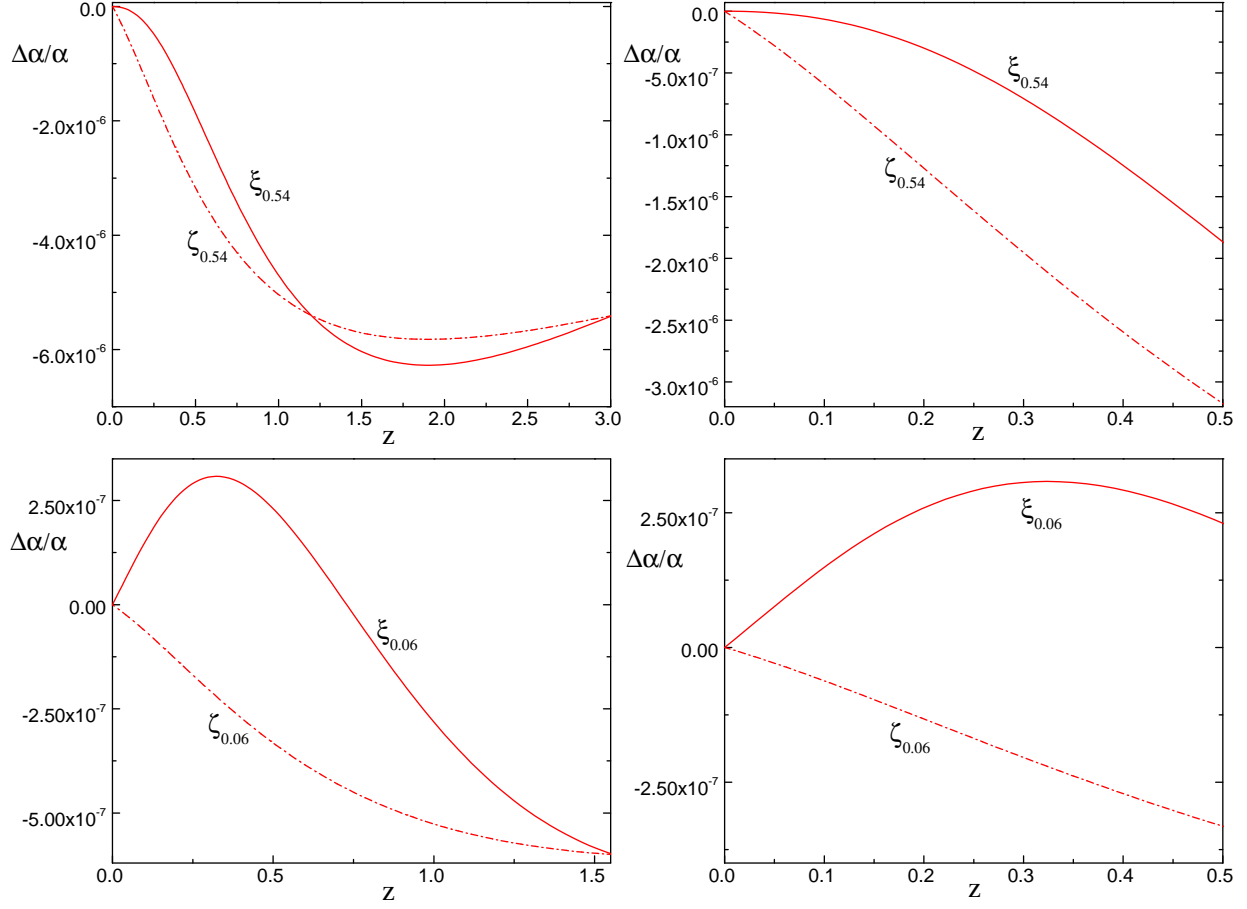
Figure 8: As in Fig. 4 for the potential (3.4) with $\lambda = 5$. In this case, $B_m \neq 0$, and we have chosen $n_m = 0.1$ and $b_m = 1$.

0.5 − 3 and zero result at $z = 1.5$, do not have to be viewed as contradictory. We also find that the early-time evolution of the field severely constrains the form of the coupling $B_F(\phi)$ with respect to its relation to $V(\phi)$, due to BBN constraints on $\Delta\alpha/\alpha$. The constraints coming from BBN are complementary to those from the late-time change in α.

Finally we have considered the case where the quintessence field is coupled to matter. Once again, BBN restricts the relationship between B and V due to the altered scaling of the matter energy density with the scale factor. Unlike the uncoupled case, the Universe briefly enters a period where the equation of state for quintessence is very stiff, before it evolves to a more standard dark energy EOS with $\omega_\phi \approx -1$. In order to have a $O(10^{-5} − 10^{-6})$ relative change of α at redshifts $z \sim 1$, these models typically require larger values of coupling between the scalar field and electromagnetic radiation.

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